Cambridge Assessment Admissions Testing

Test of Mathematics for University Admission

Paper 1 practice paper worked answers

Test of Mathematics for University Admission, Practice Paper 1
Worked Solutions

Version 1.0, May 2019

Contents

Introduction for students

These solutions are designed to support you as you prepare to take the Test of Mathematics for University Admission. They are intended to help you understand how to answer the questions, and therefore you are strongly encouraged to **attempt the questions first** before looking at these worked solutions. For this reason, each solution starts on a new page, so that you can avoid looking ahead.

The solutions contain much more detail and explanation than you would need to write in the test itself – after all, the test is multiple choice, so no written solutions are needed, and you may be very fluent at some of the steps spelled out here. Nevertheless, doing too much in your head might lead to making unnecessary mistakes, so a healthy balance is a good target!

There may be alternative ways to correctly answer these questions; these are not meant to be 'definitive' solutions.

The questions themselves are available on the 'Preparing for the test' section on the Admissions Testing website.

Question 1

We expand $(ax+b)^3$ and compare with the given expansion.

$$(ax+b)^3 = a^3x^3 + 3a^2bx^2 + 3ab^2x + b^3 = 8x^3 - px^2 + 18x - 3\sqrt{3}.$$

So we must have:

$$\begin{aligned} a^3 &= 8 \\ 3a^2b &= -p \\ 3ab^2 &= 18 \\ b^3 &= -3\sqrt{3} \end{aligned}$$

The final equation can be rewritten as $b^3 = -3^{\frac{3}{2}}$, so $b = -3^{\frac{1}{2}} = -\sqrt{3}$.

Then the third equation becomes $3a \times 3 = 18$, so $a = 2$.

Finally, the second equation gives $3 \times 2^2 \times (-\sqrt{3}) = -p$, so $p = 12\sqrt{3}$ and the answer is H.

Question 2

We can use the factor theorem to find the value of a; alternatively we can just divide by $(x+2)$ and deduce a that way, as we will need to divide by $(x+2)$ anyway to complete the factorisation. We show both methods.

The factor theorem states that in this case, $\mathrm{f}(-2) = 0$, where $\mathrm{f}(x)$ is the polynomial. So $3(-2)^3 + 13(-2)^2 + 8(-2) + a = 0$, giving $-24 + 52 - 16 + a = 0$, so $a = -12$. We can now divide the polynomial by $(x+2)$.

Alternatively, dividing the given polynomial by $(x+2)$ by considering the x^3, x^2 and x terms, but ignoring the constant term, gives:

$$3x^3 + 13x^2 + 8x + a = (x+2)(3x^2 + 7x - 6)$$

and so $a = -12$, though that is actually not needed to answer the question.

Finally, we can factorise $3x^2 + 7x - 6$ to determine the answer. Alternatively, we can use the suggested options: only options D and E give a constant term in the quadratic of -6, so it must be one of those. D expands to $3x^2 - 7x - 6$ and E expands to $3x^2 + 7x - 6$, so the answer is E.

Question 3

We need to find the gradient of the curve at $(1, 2)$. We have $y = 2x^{-2}$ and so $\frac{dy}{dx} = -4x^{-3}$. At $x = 1$, this gives $\frac{dy}{dx} = -4$ and so the normal has gradient $\frac{1}{4}$. As it passes through $(1, 2)$, it has equation

$$y - 2 = \tfrac{1}{4}(x - 1).$$

It cuts the x-axis ($y = 0$) at P, so at P, we have $-2 = \frac{1}{4}(x - 1)$, giving $x = -7$, and so P has coordinates $(-7, 0)$.

It cuts the y-axis ($x = 0$) at Q, so at Q, we have $y - 2 = \frac{1}{4}(-1)$, giving $y = \frac{7}{4}$, and so Q has coordinates $(0, \frac{7}{4})$.

Thus PQ has length

$$\sqrt{7^2 + (\tfrac{7}{4})^2} = \frac{7}{4}\sqrt{4^2 + 1} = \frac{7\sqrt{17}}{4}$$

(where we have written $7 = \frac{7}{4} \times 4$ for the first equality).

Hence the answer is C.

Question 4

__Commentary:__ This looks quite scary! It is unlikely that you have ever seen a sequence looking like this, so a sensible thing to do is to work out the first few values and look for any patterns.

We calculate the first few terms of the sequence:

$$\begin{aligned} a_1 &= (-1)^1 - (-1)^0 + (-1)^3 = (-1) - 1 + (-1) = -3 \\ a_2 &= (-1)^2 - (-1)^1 + (-1)^4 = 1 - (-1) + 1 = 3 \\ a_3 &= (-1)^3 - (-1)^2 + (-1)^5 = (-1) - 1 + (-1) = -3 \\ a_4 &= (-1)^4 - (-1)^3 + (-1)^6 = 1 - (-1) + 1 = 3 \end{aligned}$$

The pattern is now clear (and we could prove it if we wished to): the sequence goes -3, 3, -3, 3, -3, 3 and so on. So the sum of each pair of terms is zero: $a_1 + a_2 = 0$, $a_3 + a_4 = 0$, ..., $a_{37} + a_{38} = 0$. Thus

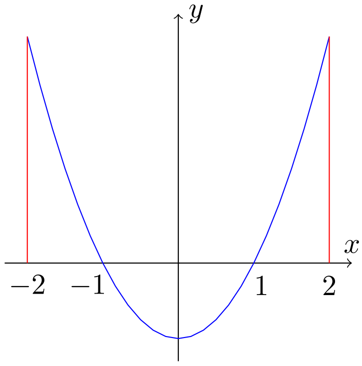
$$\begin{aligned} \sum_{n=1}^{39} a_n &= (a_1 + a_2) + (a_3 + a_4) + \cdots + (a_{37} + a_{38}) + a_{39} \\ &= 0 + 0 + \cdots + 0 + (-3) \\ &= -3 \end{aligned}$$

and the answer is B.

Question 5

To answer this question, it is worth drawing a sketch.

The graph is of $y = x^2 - 1 = (x+1)(x-1)$, so the parabola intersects the x-axis at $(1, 0)$ and $(-1, 0)$:



To find the area enclosed, we integrate over the three separate regions, from -2 to -1, from -1 to 1, and from 1 to 2:

$$\begin{aligned}
\int_{-2}^{-1} x^2 - 1 \, \mathrm{d}x &= \left[\tfrac{1}{3}x^3 - x\right]_{-2}^{-1} \\
&= \left(-\tfrac{1}{3} + 1\right) - \left(-\tfrac{8}{3} + 2\right) \\
&= \tfrac{4}{3} \\
\int_{-1}^{1} x^2 - 1 \, \mathrm{d}x &= \left[\tfrac{1}{3}x^3 - x\right]_{-1}^{1} \\
&= \left(\tfrac{1}{3} - 1\right) - \left(-\tfrac{1}{3} + 1\right) \\
&= -\tfrac{4}{3} \\
\int_{1}^{2} x^2 - 1 \, \mathrm{d}x &= \left[\tfrac{1}{3}x^3 - x\right]_{1}^{2} \\
&= \left(\tfrac{8}{3} - 2\right) - \left(\tfrac{1}{3} - 1\right) \\
&= \tfrac{4}{3}
\end{aligned}$$

Thus the three areas are each $\frac{4}{3}$, and the total area is 4, so the answer is C.

Question 6

We can either work in fractions or percentages. This solution works in fractions.

The fraction of red paint in P is $\frac{3}{10}$ and in Q is $\frac{1}{5}$. Let the fraction of red paint in R be x.

The combination is in the ratio 12 : 5 : 3, with $\frac{1}{4}$ red paint, so the composition is as follows:

Paint	Volume	Red
P	12	$12 \times \frac{3}{10}$
Q	5	$5 \times \frac{1}{5}$
R	3	$3x$
total	20	$20 \times \frac{1}{4}$

We can now sum the individual red amounts, giving:

$$12 \times \tfrac{3}{10} + 5 \times \frac{1}{5} + 3x = 20 \times 14$$

so $\frac{23}{5} + 3x = 5$, giving $3x = \frac{2}{5} = 40\%$, so $x = 13\frac{1}{3}\%$.

The correct answer is C.

Question 7

There are two factors for each member: whether the member is a man or woman, and whether the member plays tennis or cricket. We can either display this situation as a tree diagram or as a two-way table. Since the information given has both the fraction of male members who play cricket and the fraction of the cricketing members who are women, it will be simpler to use a two-way table. (In a tree diagram, we could easily display one of the two, but the other will be hard.)

We could also work with fractions or with numbers of people. In this solution, we will work with numbers. The 60% suggests that we work with a multiple of 100 people, and the presence of a fraction $\frac{2}{3}$, suggests that we work with a multiple of 3. So we start with 300 people. This is the information we obtain from the 60% of members are women and $\frac{2}{5}$ of the resulting 120 male members play cricket:

	Men	Women	Total
Tennis	72		
Cricket	48		
Total	120	180	300

We are also told that $\frac{2}{3}$ of the cricketing members are women, so the 48 cricketing men are $\frac{1}{3}$ of the cricketers, hence 96 cricketers are women, leaving 84 women to play tennis:

	Men	Women	Total
Tennis	72	84	(156)
Cricket	48	96	(144)
Total	120	180	300

(The parenthesised figures are not needed to finish the question.)

Therefore the probability that a member of the club, chosen at random, is a woman who plays tennis is $\dfrac{84}{300} = \dfrac{28}{100} = \dfrac{7}{25}$, and the answer is B.

Question 8

We note that the only angle involved is $2x$, and $0° \leqslant x \leqslant 360°$ gives $0° \leqslant 2x \leqslant 720°$.

We start by writing everything in terms of $\sin 2x$, giving:

$$(1 - \sin^2 2x) + \sqrt{3}\sin 2x - \tfrac{7}{4} = 0$$

which rearranges to give

$$\sin^2 2x - \sqrt{3}\sin 2x + \tfrac{3}{4} = 0.$$

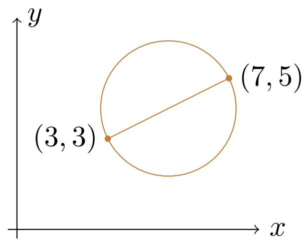
We can apply the quadratic formula to this to obtain

$$\sin 2x = \frac{\sqrt{3} \pm \sqrt{3-3}}{2} = \frac{\sqrt{3}}{2}$$

and hence the possible values of $2x$ in the range are $2x = 60°$, $120°$, $420°$ and $480°$. Thus the largest possible value of x in the range $0° \leqslant x \leqslant 360°$ is $240°$, and the answer is F.

Question 9

The initial circle looks like this:



The centre of the circle is at the midpoint of this diameter, which is $\left(\frac{3+7}{2}, \frac{3+5}{2}\right) = (5, 4)$, and the radius is therefore

$$\sqrt{(5-3)^2 + (4-3)^2} = \sqrt{5}.$$

The transformations do the following to the centre and radius of the circle:

	Centre	Radius
original	$(5, 4)$	$\sqrt{5}$
after translation	$(2, 4)$	$\sqrt{5}$
after reflection	$(2, -4)$	$\sqrt{5}$
after enlargement	$(2, -4)$	$4\sqrt{5}$

Therefore the final circle has equation

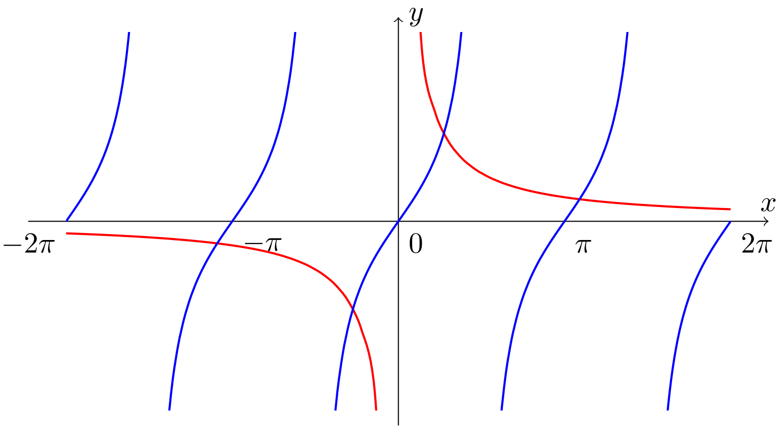
$$(x-2)^2 + (y+4)^2 = (4\sqrt{5})^2 = 80$$

and the answer is D.

Question 10

We can rearrange this equation as $\tan x = \dfrac{1}{x}$. (We note that $x = 0$ is not a solution of the original equation $x \tan x = 1$, so we can divide by x without losing any solutions.)

It is impossible to solve this equation exactly, but we are only asked for the number of solutions. Therefore we sketch the graphs of $y = \tan x$ and $y = \frac{1}{x}$ on the same axes, and count the number of intersections.



There are four points of intersection with $-2\pi \leqslant x \leqslant 2\pi$, so the answer is 4, which is option E.

Question 11

This is a quadratic in 2^{2x}, so we write $y = 2^{2x}$ and solve for y. The quadratic becomes

$$y^2 + 12 = 8y$$

as $4^{2x} = (2^2)^{2x} = 2^{4x} = (2^{2x})^2$ and $2^{2x+3} = 2^{2x} \times 2^3$.

This rearranges to $y^2 - 8y + 12 = 0$, which factorises as $(y-6)(y-2) = 0$, so $y = 6$ or $y = 2$. Thus $2^{2x} = 6$ or $2^{2x} = 2$, with the value of x being larger in the first than in the second, so $2^{2p} = 6$ and $2^{2q} = 2$. The options are given in terms of log to base 10, so we take $\log_{10}$ of the first equation. Thus $2p \log_{10} 2 = \log_{10} 6$ and $2q = 1$, and hence

$$p = \frac{\log_{10} 6}{2 \log_{10} 2} \quad \text{and} \quad q = \frac{1}{2}.$$

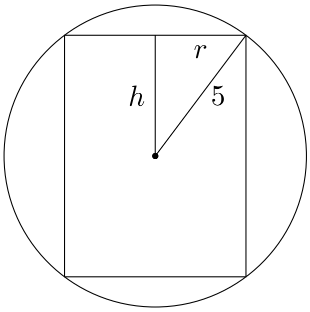
Therefore

$$\begin{aligned} p - q &= \frac{\log_{10} 6}{2\log_{10} 2} - \frac{1}{2} \\ &= \frac{\log_{10} 6 - \log_{10} 2}{2 \log_{10} 2} \\ &= \frac{\log_{10} 3}{\log_{10} 4} \end{aligned}$$

so the answer is option E.

Question 12

Let the radius of the cylinder be r. Then the diagram is as follows (with all measurements in cm), where we have drawn in two extra lines:



The 5cm is the radius of the sphere and hence of the circle shown (as the cross section is through the centre of the sphere).

Pythagoras's theorem gives $h^2 + r^2 = 5^2$, and the volume of the cylinder is $V = \pi r^2(2h) = \pi(5^2 - h^2)(2h) = 2\pi(25h - h^3)$. We can maximise this by differentiating with respect to h. (We could write everything in terms of r instead, but that would involve square roots.)

We have $\dfrac{\mathrm{d}V}{\mathrm{d}h} = 2\pi(25 - 3h^2)$, which is zero when $25 = 3h^2$, so $h = \dfrac{5}{\sqrt{3}}$. Substituting this into the formula for V gives the largest possible V as

$$V = 2\pi(5^2 - h^2)h = 2\pi\left(25 - \frac{25}{3}\right)\frac{5}{\sqrt{3}} = \frac{500}{3\sqrt{3}}\pi = \frac{500\sqrt{3}}{9}\pi$$

hence the answer is E.

Question 13

Commentary: *This is not something that you will have learnt how to do, and in general, this is a very difficult problem indeed. But we can use the tools at our disposal to do something helpful. We don't know how to solve a quintic (x^5) equation, but we can at least try to find the turning (stationary) points, and see where that gets us.*

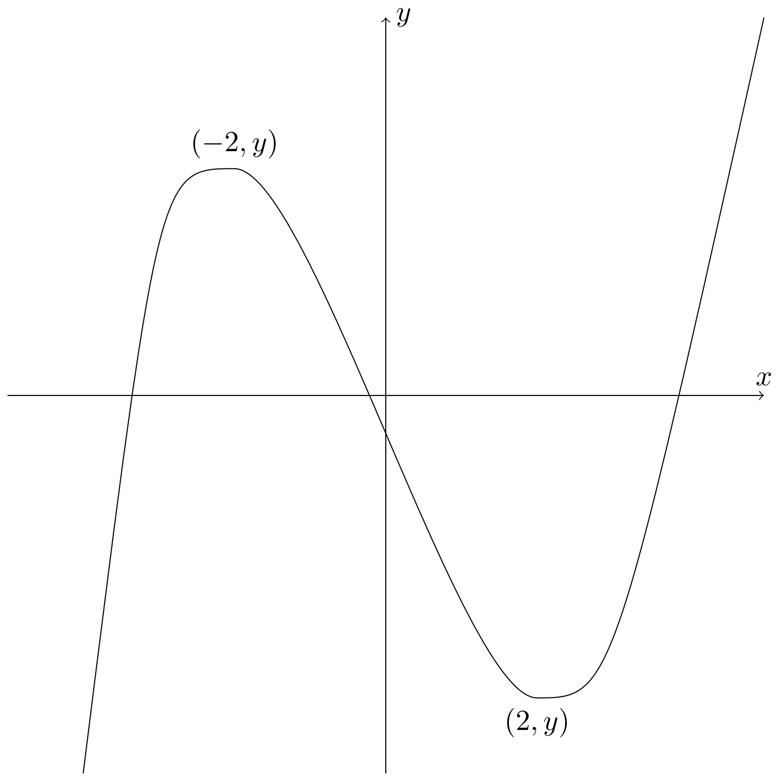
We attempt to sketch the graph of $y = 3x^5 - 10x^3 - 120x + 30$, or at least think about what a sketch might look like.

We calculate

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 15x^4 - 30x^2 - 120 = 15(x^4 - 2x^2 - 8) = 15(x^2 - 4)(x^2 + 2)$$

so there are stationary points when $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$, that is, when $x = 2$ and when $x = -2$. We can now calculate the y-coordinates: when $x = -2$, $y = -36 + 80 + 240 + 30 > 0$ (we do not need to calculate the exact value), and when $x = 2$, $y = 36 - 80 - 240 + 30 < 0$.

So the graph must look something like the following:



There are clearly 3 points of intersection of this graph with $y = 0$, so the original equation has 3 real roots, and the answer is C.

Question 14

Let us write out the first few terms of the three sequences explicitly. Let the common ratio of T be R and the common ratio of U be r.

term	1	2	3	4	5	...	n	...
T	4	$4R$	$4R^2$	$4R^3$	$4R^4$	...	$4R^{n-1}$	...
U	4	$4r$	$4r^2$	$4r^3$	$4r^4$	...	$4r^{n-1}$	...
S	8	$4R+4r$	$4R^2+4r^2$	$4R^3+4r^3$	$4R^4+4r^4$	...	$4R^{n-1}+4r^{n-1}$	...

We are given the first three terms of S, so

$$\begin{aligned} 8 &= 8 \\ 4R+4r &= 3 \\ 4R^2+4r^2 &= \tfrac{5}{4} \end{aligned}$$

Therefore $R+r=\frac{3}{4}$ and $R^2+r^2=\frac{5}{16}$. We can work out the values of R and r by guessing, or we can work them out by solving these equations. We show the second approach here.

The first equation gives $R=\frac{3}{4}-r$, which we can substitute into the second equation to give

$$(\tfrac{3}{4}-r)^2+r^2=\tfrac{5}{16}$$

so $2r^2-\frac{3}{2}r+\frac{1}{4}=0$, hence $8r^2-6r+1=0$, which factorises as $(4r-1)(2r-1)=0$, giving $r=\frac{1}{2}$ or $r=\frac{1}{4}$ and $R=\frac{1}{4}$ or $R=\frac{1}{2}$ respectively.

We can therefore work out the sums to infinity of T and U, taking $R=\frac{1}{4}$ and $r=\frac{1}{2}$:

$$\begin{aligned} \text{sum to infinity of T} &= \frac{4}{1-\frac{1}{4}} = \tfrac{16}{3} \\ \text{sum to infinity of U} &= \frac{4}{1-\frac{1}{2}} = 8 \end{aligned}$$

giving the sum to infinity of S as $\frac{16}{3}+8=\frac{40}{3}$, since S is the sum of T and U. If we took the values of R and r to be the other way round, we would have the series T and U swapped, but their sum would be unchanged.

So the answer is D.

Question 15

We start by expanding the brackets to get

$$y = (2x + a)(x^2 - 4ax + 4a^2) = 2x^3 - 7ax^2 + 4a^2x + 8a^3$$

so the derivative is given by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 6x^2 - 14ax + 4a^2.$$

Therefore at $x = 1$, the gradient is $6 - 14a + 4a^2$. To find the least possible value of this as a varies, we can either differentiate this expression with respect to a or complete the square. The latter approach gives

$$\begin{aligned} 4a^2 - 14a + 6 &= 4(a^2 - \tfrac{7}{2}a) + 6 \\ &= 4\big((a - \tfrac{7}{4})^2 - (\tfrac{7}{4})^2\big) + 6 \\ &= 4(a - \tfrac{7}{4})^2 - \tfrac{49}{4} + 6 \\ &= 4(a - \tfrac{7}{4})^2 - \tfrac{25}{4} \end{aligned}$$

so the minimum value is $-\frac{25}{4}$ and the answer is C.

Question 16

The first equation involves the sum of two logarithms, so we can rewrite it as:

$$\log_{10}(2(y-1)) = \log_{10} x^2$$

and so $2(y-1) = x^2$.

The second equation becomes $y + 3 - 3x = 1$, as $\log_a 1 = 0$ for any base a.

We therefore have two simultaneous equations:

$$\begin{aligned} 2y - 2 &= x^2 \\ y - 3x + 2 &= 0. \end{aligned}$$

We want to find the values of y, so we rewrite the second equation to eliminate x:

$$x = \frac{y+2}{3}$$

and so the first equation becomes

$$2y - 2 = \frac{(y+2)^2}{9}$$

or

$$18y - 18 = y^2 + 4y + 4$$

which rearranges to give the quadratic $y^2 - 14y + 22 = 0$. The solutions to this quadratic are

$$\frac{14 \pm \sqrt{14^2 - 4 \times 22}}{2} = 7 \pm \sqrt{7^2 - 22} = 7 \pm \sqrt{27}$$

and so the correct answer is C.

It would be good to check whether both of these solutions are valid (though the question does not require us to do so). To be valid, we require both $x > 0$ and $y > 1$. Since $\sqrt{27} < 6$, we see that $y > 1$. Also, as $x = \frac{1}{3}(y+2)$ and $y > 0$, it follows that $x > 0$. So there are, indeed, two valid solutions.

Question 17

The formula for y is the product of two factors, namely $1 + 2\cos x$ and $\cos 2x$. The whole expression is negative when one of the two factors is positive and the other is negative. The factors change sign when they cross a point where the factor is zero, and we can find these points:

- $1 + 2\cos x = 0$ when $\cos x = -\frac{1}{2}$, which is when $x = \frac{2\pi}{3}$ (within the range $0 < x < \pi$).
- $\cos 2x = 0$ when $x = \frac{\pi}{4}$ and when $x = \frac{3\pi}{4}$.

We can now make a table showing the signs of the two factors in the different parts of the interval $0 < x < \pi$. (This is a useful technique in general.)

	$0 < x < \frac{\pi}{4}$	$x = \frac{\pi}{4}$	$\frac{\pi}{4} < x < \frac{2\pi}{3}$	$x = \frac{2\pi}{3}$	$\frac{2\pi}{3} < x < \frac{3\pi}{4}$	$x = \frac{3\pi}{4}$	$\frac{3\pi}{4} < x < \pi$
$1 + 2\cos x$	$+$	$+$	$+$	0	$-$	$-$	$-$
$\cos 2x$	$+$	0	$-$	$-$	$-$	0	$+$
y	$+$	0	$-$	0	$+$	0	$-$

Therefore y is negative when $\frac{\pi}{4} < x < \frac{2\pi}{3}$ and when $\frac{3\pi}{4} < x < \pi$, and so the answer is D.

Question 18

This requires us to first differentiate the function. We therefore write

$$y = \frac{1-x}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}} - x^{\frac{1}{3}}$$

which we can differentiate to get

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\tfrac{2}{3}x^{-\frac{5}{3}} - \tfrac{1}{3}x^{-\frac{2}{3}}$$

This is zero when

$$-\tfrac{2}{3}x^{-\frac{5}{3}} - \tfrac{1}{3}x^{-\frac{2}{3}} = 0,$$

so multiplying by $3x^{\frac{5}{3}}$ to clear fractions in both the coefficients and in the powers gives

$$-2 - x = 0$$

so $x = -2$. (We could also have obtained this by writing $-\frac{2}{3}x^{-\frac{5}{3}} = \frac{1}{3}x^{-\frac{2}{3}}$ and dividing one side by the other.)

We next need to determine the sign of $\frac{\mathrm{d}y}{\mathrm{d}x}$ in each region. We note that the function is not defined at $x = 0$, so we have to deal with $x < 0$ and $x > 0$ separately. It is also not clear how to find the sign of $\frac{\mathrm{d}y}{\mathrm{d}x}$ directly from its given form, so we first factorise it, giving

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\tfrac{1}{3}x^{-\frac{5}{3}}(2+x).$$

(Incidentally, this gives yet another way to see that the derivative is zero at $x = -2$.) We can now work out the signs of the two factors, and hence of the derivative, in the various ranges:

	$x < -2$	$x = -2$	$-2 < x < 0$	$x > 0$
$x^{-\frac{5}{3}}$	$-$	$-$	$-$	$+$
$2 + x$	$-$	0	$+$	$+$
$\frac{\mathrm{d}y}{\mathrm{d}x}$	$+$	0	$-$	$+$

It is therefore increasing in the region $x < -2$ and $x > 0$. (It it not increasing at $x = -2$, but rather it is stationary at that point.) The correct answer is therefore A (though the question mistakenly says $x \leqslant -2$).

Question 19

We expand the first expression up to powers of x^3 to find its coefficient. We can think of the first expression as $(1 + (2x + 3x^2))^6$ and use the binomial theorem to expand it. We note that the first few binomial coefficients are

$$\binom{6}{0} = 1; \qquad \binom{6}{1} = 6; \qquad \binom{6}{2} = \frac{6 \times 5}{2!} = 15; \qquad \binom{6}{3} = \frac{6 \times 5 \times 4}{3!} = 20.$$

We thus have

$$\begin{aligned}(1 + (2x + 3x^2))^6 &= 1 + \tbinom{6}{1}(2x + 3x^2) + \tbinom{6}{2}(2x + 3x^2)^2 + \tbinom{6}{3}(2x + 3x^2)^3 + \cdots \\ &= 1 + 6(2x + 3x^2) + 15((2x)^2 + 2(2x)(3x^2) + \cdots) + 20((2x)^3 + \cdots) + \cdots\end{aligned}$$

where we have stopped when the powers reach 3. We can read off the coefficient of x^3 from this; it is

$$15 \times 2 \times 2 \times 3 + 20 \times 8 = 340.$$

We can expand $(1 - ax^2)^5$ similarly, and obtain

$$1 + \tbinom{5}{1}(-ax^2) + \tbinom{5}{2}(-ax^2)^2 + \cdots = 1 - 5ax^2 + 10a^2x^4 + \cdots$$

so the coefficient of x^4 is $10a^2$.

We are told how these two coefficients relate to each other: we have

$$340 = 2 \times 10a^2$$

so $a^2 = 17$ and $a = \pm\sqrt{17}$, giving the answer as option B.

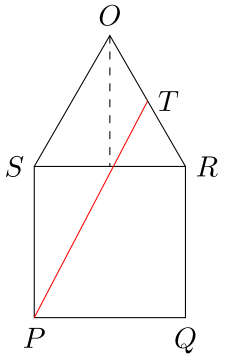
Question 20

Let us label the midpoint of OR as T, so that we can refer to it.

We first observe that since all the edges of the pyramid are of length 20m, the triangles are all equilateral.

There are two ways to get from P to the midpoint of OR in a short distance: one can either travel along the base to RS and then along the triangular face to T, or one could travel along the triangular face OPS to the edge OS and from there to T. It is not obvious which route is shorter, so we will calculate both. (There are also routes which are mirror-reflections of these going via the edge QR or OQ.)

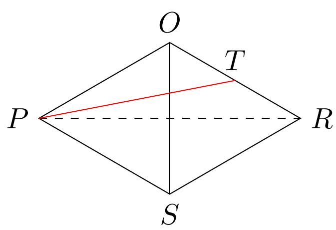
To find these distances, we can "unfold" the tetrahedron, that is, make a net for the tetrahedron. For the first route, we draw just the faces $PQRS$ and ORS:



The height of the triangle OSR is $20\sin 60^\circ = 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}$, so T lies $5\sqrt{3}$ above the line SR; T also lies $\frac{3}{4} \times 20 = 15$ to the right of S (as it is at the midpoint of OR). We can therefore find the length PT using Pythagoras's theorem:

$$\begin{aligned} PT^2 &= 15^2 + (20 + 5\sqrt{3})^2 \\ &= 225 + 400 + 200\sqrt{3} + 75 \\ &= 700 + 200\sqrt{3} \end{aligned}$$

We now look at the other route, which we can do by drawing the triangles OPS and OSR:



Using the height of the triangle as calculated above, P lies $10\sqrt{3}$ to the left of OS and T lies $5\sqrt{3}$ to its right, while T lies 5 above P (being a quarter of OS). Pythagoras's theorem therefore gives

$$\begin{aligned} PT^2 &= (15\sqrt{3})^2 + 5^2 \\ &= 225 \times 3 + 25 \\ &= 700 \end{aligned}$$

This is therefore the shorter of the two routes, and its length is $\sqrt{700} = 10\sqrt{7}$, hence the solution is option D.

We are Cambridge Assessment Admissions Testing, part of the University of Cambridge. Our research-based tests provide a fair measure of skills and aptitude to help you make informed decisions. As a trusted partner, we work closely with universities, governments and employers to enhance their selection processes.

Cambridge Assessment
Admissions Testing
The Triangle Building
Shaftesbury Road
Cambridge
CB2 8EA
United Kingdom

Admissions tests support:
admissionstesting.org/help